

# Math Formulas: Integrals of Rational Functions

## Integrals involving $ax + b$

1. 
$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)}, \quad (\text{for } n \neq -1)$$
2. 
$$\int \frac{1}{ax + b} dx = \frac{1}{a} \ln |ax + b|$$
3. 
$$\int x(ax + b)^n dx = \frac{a(n+1)x - b}{a^2(n+1)(n+2)} (ax + b)^{n+1}, \quad (\text{for } n \neq -1, n \neq -2)$$
4. 
$$\int \frac{x}{ax + b} dx = \frac{x}{2} - \frac{b}{a^2} \ln |ax + b|$$
5. 
$$\int \frac{x}{(ax + b)^2} dx = \frac{b}{a^2(ax + b)} - \frac{1}{a^2} \ln |ax + b|$$
6. 
$$\int \frac{x^2}{ax + b} dx = \frac{1}{a^3} \left( \frac{(ax + b)^2}{2} - 2b(ax + b) + b^2 \ln |ax + b| \right)$$
7. 
$$\int \frac{x^2}{(ax + b)^2} dx = \frac{1}{a^3} \left( ax + b - 2b \ln |ax + b| - \frac{b^2}{ax + b} \right)$$
8. 
$$\int \frac{x^2}{(ax + b)^3} dx = \frac{1}{a^3} \left( \ln |ax + b| + \frac{2b}{ax + b} - \frac{b^2}{2(ax + b)^2} \right)$$
9. 
$$\int \frac{x^2}{(ax + b)^n} dx = \frac{1}{a^3} \left( -\frac{(ax + b)^{3-n}}{n-3} + \frac{2b(ax + b)^{2-n}}{n-2} - \frac{b^2(ax + b)^{1-n}}{n-1} \right)$$
10. 
$$\int \frac{1}{x(ax + b)} dx = -\frac{1}{b} \ln \left| \frac{ax + b}{x} \right|$$
11. 
$$\int \frac{1}{x^2(ax + b)^2} dx = -\frac{1}{bx} + \frac{a}{b^2} \ln \left| \frac{ax + b}{x} \right|$$
12. 
$$\int \frac{1}{x^2(ax + b)^2} dx = -a \left( \frac{1}{b^2(ax + b)} + \frac{1}{ab^2x} - \frac{2}{b^3} \ln \left| \frac{ax + b}{x} \right| \right)$$

## Integrals involving $ax^2 + bx + c$

13. 
$$\frac{1}{x^2 + a^2} dx = \frac{1}{a} \arctan \frac{x}{a}$$
14. 
$$\frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \left| \frac{x - a}{x + a} \right|$$
15. 
$$\int \frac{1}{ax^2 + bx + c} dx = \begin{cases} \frac{2}{\sqrt{4ac - b^2}} \arctan \frac{2ax + b}{\sqrt{4ac - b^2}} & \text{for } 4ac - b^2 > 0 \\ \frac{2}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}} \right| & \text{for } 4ac - b^2 < 0 \\ -\frac{2}{2ax + b} & \text{for } 4ac - b^2 = 0 \end{cases}$$
16. 
$$\int \frac{x}{ax^2 + bx + c} dx = \frac{1}{2a} \ln |ax^2 + bx + c| - \frac{b}{2a} \int \frac{dx}{ax^2 + bx + c}$$
17. 
$$\int \frac{1}{(ax^2 + bx + c)^n} dx = \frac{2ax + b}{(n-1)(4ac - b^2)(ax + bx + c)^{n-1}} + \frac{2(2n-3)a}{(n-1)(4ac - b^2)} \int \frac{dx}{(ax^2 + bx + c)^{n-1}}$$

18. 
$$\int \frac{1}{x(ax^2 + bx + c)} dx = \frac{1}{2c} \ln \left| \frac{x^2}{ax^2 + bx + c} \right| - \frac{b}{2c} \int \frac{1}{ax^2 + bx + c} dx$$