

Math Formulas: Hyperbolic functions

Definitions of hyperbolic functions

1. $\sinh x = \frac{e^x - e^{-x}}{2}$
2. $\cosh x = \frac{e^x + e^{-x}}{2}$
3. $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh x}{\cosh x}$
4. $\operatorname{csch} x = \frac{2}{e^x - e^{-x}} = \frac{1}{\sinh x}$
5. $\operatorname{sech} x = \frac{2}{e^x + e^{-x}} = \frac{1}{\cosh x}$
6. $\operatorname{coth} x = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{\cosh x}{\sinh x}$

Derivatives

7. $\frac{d}{dx} \sinh x = \cosh x$
8. $\frac{d}{dx} \cosh x = \sinh x$
9. $\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$
10. $\frac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \cdot \coth x$
11. $\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \cdot \tanh x$
12. $\frac{d}{dx} \operatorname{coth} x = -\operatorname{csch}^2 x$

Hyperbolic identities

13. $\cosh^2 x - \sinh^2 x = 1$
14. $\tanh^2 x + \operatorname{sech}^2 x = 1$
15. $\coth^2 x - \operatorname{csch}^2 x = 1$
16. $\sinh(x \pm y) = \sinh x \cdot \cosh y \pm \cosh x \cdot \sinh y$
17. $\cosh(x \pm y) = \cosh x \cdot \cosh y \pm \sinh x \cdot \sinh y$
18. $\sinh(2 \cdot x) = 2 \cdot \sinh x \cdot \cosh x$
19. $\cosh(2 \cdot x) = \cosh^2 x + \sinh^2 x$

$$20. \quad \sinh^2 x = \frac{-1 + \cosh 2x}{2}$$

$$21. \quad \cosh^2 x = \frac{1 + \cosh 2x}{2}$$

Inverse Hyperbolic functions

$$22. \quad \sinh^{-1} x = \ln \left(x + \sqrt{x^2 + 1} \right), \quad x \in (-\infty, \infty)$$

$$23. \quad \cosh^{-1} x = \ln \left(x + \sqrt{x^2 - 1} \right), \quad x \in [1, \infty)$$

$$24. \quad \tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right), \quad x \in (-1, 1)$$

$$25. \quad \coth^{-1} x = \frac{1}{2} \ln \left(\frac{x+1}{x-1} \right), \quad x \in (-\infty, -1) \cup (1, \infty)$$

$$26. \quad \operatorname{sech}^{-1} x = \ln \left(\frac{1 + \sqrt{1-x^2}}{x} \right), \quad x \in (0, 1]$$

$$27. \quad \operatorname{csch}^{-1} x = \ln \left(\frac{1}{x} + \frac{\sqrt{1-x^2}}{|x|} \right), \quad x \in (-\infty, 0) \cup (0, \infty)$$

Derivatives of Inverse Hyperbolic functions

$$28. \quad \frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{x^2 + 1}}$$

$$29. \quad \frac{d}{dx} \cosh^{-1} x = \frac{1}{\sqrt{x^2 - 1}}$$

$$30. \quad \frac{d}{dx} \tanh^{-1} x = \frac{1}{1-x^2}$$

$$31. \quad \frac{d}{dx} \operatorname{csch}^{-1} x = -\frac{1}{|x|\sqrt{1+x^2}}$$

$$32. \quad \frac{d}{dx} \operatorname{sech}^{-1} x = -\frac{1}{x\sqrt{1-x^2}}$$

$$33. \quad \frac{d}{dx} \coth^{-1} x = \frac{1}{1-x^2}$$