Math Formulas: Solutions of algebraic equations

Quadric Equation: $ax^2 + bx + c = 0$

Solutions (roots):

1. $$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $D = b^2 - 4ac$ is the **discriminant** , then the roots are

1. real and unique if $D > 0$

2. real and equal if $D = 0$

3. complex conjugate if $D < 0$

Cubic Equation: $x^3 + a_1x^2 + a_2x + a_3 = 0$

Let

2. $$\begin{aligned} Q &= \frac{3a_2 - a_1^2}{9} \\ R &= \frac{9a_1a_2 - 27a_3 - 2a_1^3}{54} \\ S &= \sqrt[3]{R + \sqrt{Q^3 + R^2}} \\ T &= \sqrt[3]{R - \sqrt{Q^3 + R^2}} \end{aligned}$$

Then **solutions (roots)** of the cubic equation are:

3. $$\begin{aligned} x_1 &= S + T - \frac{1}{3}a_1 \\ x_2 &= -\frac{1}{2}(S+T) - \frac{1}{3}a_1 + \frac{1}{2}\,i\,\sqrt{3}(S-T) \\ x_3 &= -\frac{1}{2}(S+T) - \frac{1}{3}a_1 - \frac{1}{2}\,i\,\sqrt{3}(S-T) \end{aligned}$$

If $D = Q^3 + R^2$ is the **discriminant** of the cubic equation, then:

1. one root is real and two complex conjugate if $D > 0$

2. all roots are real and at last two are equal if $D = 0$

3. all roots are real and unequal if $D < 0$

Quartic Equation:$x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$

Let y_1 be a real root of the cubic equation

4. $$y^3 - a_2y^2 + (a_1a_3 - 4a_4)y + (4a_2a_4 - a_3^2 - a_1^2a_4) = 0$$

¡p class=”zu”¿ Then **solutions** of the quartic equation are the 4 roots of

5. $$z^2 + \frac{1}{2}\left(a_1 \pm \sqrt{a_1^2 - 4a_2 + 4y_1}\right)z + \frac{1}{2}\left(y_1 \pm \sqrt{y_1^2 - 4a_4}\right) = 0$$