

Math Formulas: Definite integrals of exponential functions

1.
$$\int_0^{\infty} e^{-ax} \cos bx \, dx = \frac{a}{a^2 + b^2}$$
2.
$$\int_0^{\infty} e^{-ax} \sin bx \, dx = \frac{b}{a^2 + b^2}$$
3.
$$\int_0^{\infty} \frac{e^{-ax} \sin bx}{x} \, dx = \arctan \frac{b}{a}$$
4.
$$\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x} \, dx = \ln \frac{b}{a}$$
5.
$$\int_0^{\infty} e^{-ax^2} \, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}$$
6.
$$\int_0^{\infty} e^{-ax^2} \cos bx \, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{-\frac{b^2}{4a}}$$
7.
$$\int_{-\infty}^{\infty} e^{-(ax^2+bx+c)} \, dx = \sqrt{\frac{\pi}{a}} e^{\frac{b^2-4ac}{4a}}$$
8.
$$\int_0^{\infty} x^n e^{-ax} \, dx = \frac{\Gamma(n+1)}{a^{n+1}}$$
9.
$$\int_0^{\infty} x^m e^{-ax^2} \, dx = \frac{\Gamma\left(\frac{m+1}{2}\right)}{2a^{(m+1)/2}}$$
10.
$$\int_0^{\infty} e^{-(ax^2+b/x^2)} \, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{-2\sqrt{ab}}$$
11.
$$\int_0^{\infty} \frac{x \, dx}{e^x - 1} = \frac{\pi^2}{6}$$
12.
$$\int_0^{\infty} \frac{x^{n-1}}{e^x - 1} \, dx = \Gamma(n) \left(\frac{1}{1^n} + \frac{1}{2^n} + \frac{1}{3^n} + \dots \right)$$
13.
$$\int_0^{\infty} \frac{x \, dx}{e^x + 1} = \frac{\pi^2}{12}$$
14.
$$\int_0^{\infty} \frac{x^{n-1}}{e^x + 1} \, dx = \Gamma(n) \left(\frac{1}{1^n} - \frac{1}{2^n} + \frac{1}{3^n} - \dots \right)$$
15.
$$\int_0^{\infty} \frac{\sin mx}{e^{2\pi x} - 1} \, dx = \frac{1}{4} \coth \frac{m}{2} - \frac{1}{2m}$$
16.
$$\int_0^{\infty} \left(\frac{1}{1+x} - e^{-x} \right) \frac{dx}{x} = \gamma$$
17.
$$\int_0^{\infty} \frac{e^{-x^2} - e^{-x}}{x} \, dx = \frac{1}{2} \gamma$$
18.
$$\int_0^{\infty} \left(\frac{1}{e^x - 1} - \frac{e^{-x}}{x} \right) \, dx = \gamma$$
19.
$$\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x \sec(px)} \, dx = \frac{1}{2} \ln \left(\frac{b^2 + p^2}{a^2 + p^2} \right)$$
20.
$$\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x \csc(px)} \, dx = \arctan \frac{b}{p} - \arctan \frac{a}{p}$$
21.
$$\int_0^{\infty} \frac{e^{-ax}(1 - \cos x)}{x^2} \, dx = \operatorname{arccot} a - \frac{a}{2} \ln(a^2 + 1)$$