Functions Formulas

1. Exponents

$a^p = \underbrace{a \cdot a \cdot \ldots \cdot a}_{p}$ *if* $p \in N \;\; p > 0,\; a \in R$

$a^0 = 1$ *if* $a \neq 0$

$a^r \cdot a^s = a^{r+s}$

$\frac{a^r}{a^s} = a^{r-s}$

$\left(a^r\right)^s = a^{r \cdot s}$

$(a \cdot b)^r = a^r \cdot b^r$

$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$

$a^{-r} = \frac{1}{a^r}$

$a^{\frac{r}{s}} = \sqrt[s]{a^r}$

2. Logarithms

Definition:

$y = \log_a x \Leftrightarrow a^y = x \quad (a, x > 0,\; y \in R)$

Formulas:

$\log_a 1 = 0$

$\log_a a = 1$

$\log_a mn = \log_a m + \log_a n$

$\log_a \frac{m}{n} = \log_a m - \log_a n$

$\log_a m^n = n \log_a m$

$\log_a m = \log_b m \cdot \log_a b$

$\log_a m = \frac{\log_b m}{\log_b a}$

$\log_a b = \frac{1}{\log_b a}$

$\log_a x = \frac{\ln x}{\ln a} = (\log_a e) \ln x$

3. Roots

Definitions:

a,b: bases ($a, b \geq 0$ *if* $n = 2k$)

n,m: powers

Formulas:

$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[n]{a}\sqrt[m]{b} = \sqrt[nm]{a^m b^n}$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}},\; b \neq 0$

$\frac{\sqrt[n]{a}}{\sqrt[m]{b}} = \sqrt[nm]{\frac{a^m}{b^n}},\; b \neq 0$

$\left(\sqrt[n]{a^m}\right)^p = \sqrt[n]{a^{mp}}$

$\left(\sqrt[n]{a}\right)^n = a$

$\sqrt[n]{a^m} = \sqrt[np]{a^{mp}}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

$\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

$\frac{1}{\sqrt[n]{a}} = \frac{\sqrt[n]{a^{n-1}}}{a},\; a \neq 0$

$\sqrt{a \pm \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \pm \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}$

$\frac{1}{\sqrt{a} \pm \sqrt{b}} = \frac{\sqrt{a} \mp \sqrt{b}}{a - b}$

4. Trigonometry

Right-Triangle Definitions

$$\sin\alpha = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos\alpha = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$tg\alpha = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\csc\alpha = \frac{1}{\sin\alpha} = \frac{\text{Hypotenuse}}{\text{Opposite}}$$

$$\cot\alpha = \frac{1}{tg\alpha} = \frac{\text{Adjacent}}{\text{Opposite}}$$

$$\sec\alpha = \frac{1}{\cos\alpha} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$$

Reduction Formulas

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\sin(\frac{\pi}{2} - x) = \cos x$$

$$\cos(\frac{\pi}{2} - x) = \sin x$$

$$\sin(\frac{\pi}{2} + x) = \cos x$$

$$\cos(\frac{\pi}{2} - x) = -\sin x$$

$$\sin(\pi - x) = \sin x$$

$$\cos(\pi - x) = -\cos x$$

$$\sin(\pi + x) = -\sin x$$

$$\cos(\pi + x) = -\cos x$$

Identities

$$\sin^2 x + \cos^2 x = 1$$

$$tg^2 x + 1 = \frac{1}{\cos^2 x}$$

$$\cot^2 x + 1 = \frac{1}{\sin^2 x}$$

Sum and Difference Formulas

$$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \sin\beta\cdot\cos\alpha$$

$$\sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \sin\beta\cdot\cos\alpha$$

$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\cdot\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\cdot\tan\beta}$$

Double Angle and Half Angle Formulas

$$\sin(2\alpha) = 2\sin\alpha\cdot\cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

$$\tan(2\alpha) = \frac{2tg\alpha}{1 - tg^2\alpha}$$

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$$

Other Useful Trig Formulae

Law of sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Area of triangle

$$K = \frac{1}{2}ab\sin\gamma$$

5. Hyperbolic functions

Definitions:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh x}{\cosh x}$$

$$\operatorname{csch} x = \frac{2}{e^x - e^{-x}} = \frac{1}{\sinh x}$$

$$\operatorname{sech} x = \frac{2}{e^x + e^{-x}} = \frac{1}{\cosh x}$$

$$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{\cosh x}{\sinh x}$$

Derivates

$$\frac{d}{dx}\sinh x = \cosh x$$

$$\frac{d}{dx}\cosh x = \sinh x$$

$$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$$

$$\frac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x \cdot \coth x$$

$$\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \cdot \tanh x$$

$$\frac{d}{dx}\coth x = -\operatorname{csch}^2 x$$

Hyperbolic identities

$$\cosh^2 x - \sinh^2 x = 1$$

$$\tanh^2 x + \operatorname{sech}^2 x = 1$$

$$\coth^2 x - \operatorname{csch}^2 x = 1$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\sinh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\sinh^2 x = \frac{-1 + \cosh 2x}{2}$$

$$\cosh^2 x = \frac{1 + \cosh 2x}{2}$$

Inverse Hyperbolic functions

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right) \quad x \in (-\infty, \infty)$$

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right) \quad x \in [1, \infty)$$

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad x \in (-1, 1)$$

$$\coth^{-1} x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right) \quad x \in (-\infty, -1) \cup (1, \infty)$$

$$\operatorname{sech}^{-1} x = \ln\left(\frac{1 + \sqrt{1 - x^2}}{x}\right) \quad x \in (0, 1]$$

$$\operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1 - x^2}}{|x|}\right) \quad x \in (-\infty, 0) \cup (0, \infty)$$

Inverse Hyperbolic derivates

$$\frac{d}{dx}\sinh^{-1} x = \frac{1}{\sqrt{x^2 + 1}}$$

$$\frac{d}{dx}\cosh^{-1} x = \frac{1}{\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}\tanh^{-1} x = \frac{1}{1 - x^2}$$

$$\frac{d}{dx}\operatorname{csch} x = -\frac{1}{|x|\sqrt{1 + x^2}}$$

$$\frac{d}{dx}\operatorname{sech}^{-1} x = -\frac{1}{x\sqrt{1 - x^2}}$$

$$\frac{d}{dx}\coth^{-1} x = \frac{1}{1 - x^2}$$