Algebra Formulas

1. Set identities

Definitions:

I: Universal set
A': Complement
Empty set: $\varnothing$

Union of sets

$$A \cup B = \{x \mid x \in A \ \ or \ \ x \in B\}$$

Intersection of sets

$$A \cap B = \{x \mid x \in A \ \ and \ \ x \in B\}$$

Complement

$$A' = \{x \in I \mid x \in A\}$$

Difference of sets

$$B \setminus A = \{x \mid x \in B \ \ and \ \ x \notin A\}$$

Cartesian product

$$A \times B = \{(x, y) \mid x \in A \ and \ y \in B\}$$

Set identities involving union

Commutativity

$$A \cup B = B \cup A$$

Associativity

$$A \cup (B \cup C) = (A \cup B) \cup C$$

Idempotency

$$A \cup A = A$$

Set identities involving intersection

commutativity

$$A \cap B = B \cap A$$

Associativity

$$A \cap (B \cap C) = (A \cap B) \cap C$$

Idempotency

$$A \cap A = A$$

Set identities involving union and intersection

Distributivity

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Domination

$$A \cap \varnothing = \varnothing$$
$$A \cup I = I$$

Identity

$$A \cup \varnothing = A$$
$$A \cap I = A$$

Set identities involving union, intersection and complement

complement of intersection and union

$$A \cup A' = I$$
$$A \cap A' = \varnothing$$

De Morgan's laws

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

Set identities involving difference

$$B \setminus A = B(A \cup B)$$
$$B \setminus A = B \cap A'$$
$$A \setminus A = \varnothing$$
$$(A \setminus B) \cap C = (A \cap C) \setminus (B \cap C)$$
$$A' = I \setminus A$$

2. Sets of Numbers

Definitions:

N: Natural numbers
N_o: Whole numbers
Z: Integers
Z^+: Positive integers
Z^-: Negative integers
Q: Rational numbers
C: Complex numbers

Natural numbers (counting numbers)

$$N = \{1, \ 2, \ 3, \dots\}$$

Whole numbers (counting numbers + zero)

$$N_o = \{0, \ 1, \ 2, \ 3, \dots\}$$

Integers

$$Z^+ = N = \{1, \ 2, \ 3, \dots\}$$

$$Z^- = \{\dots, \ -3, \ -2, \ -1\}$$

$$Z = Z^- \cup \{0\} \cup Z = .\{\dots, -3, \ -2, \ -1, \ 0, \ 1, \ 2, \ 3, \dots\}$$

Irrational numbers:

Nonerepeating and nonterminating integers

Real numbers:

Union of rational and irrational numbers

Complex numbers:

$$C = \{x + iy \mid x \in R \ \ and \ \ y \in R\}$$

$$N \subset Z \subset Q \subset R \subset C$$

3. Complex numbers

Definitions:

A complex nuber is written as a + bi where a and b are real numbers an i, called the imaginary unit, has the property that $i^2=-1$.

The complex numbers a+bi and a-bi are called complex conjugate of each other.

Equality of complex numbers

a + bi = c + di if and only if a = c and b = d

Addition of complex numbers

(a + bi) + (c + di) = (a + c) + (b + d)i

Subtraction of complex numbers

(a + bi) - (c + di) = (a - c) + (b - d)i

Multiplication of complex numbers

(a + bi)(c + di) = (ac - bd) + (ad + bc)i

Division of complex numbers

$$\frac{a+bi}{c+di} = \frac{a+bi}{c+di} \cdot \frac{c-di}{c-di} = \frac{ac+bd}{c^2+d^2} + \left(\frac{bc-ad}{c^2+d^2}\right)i$$

Polar form of complex numbers

$$x + iy = r(\cos\theta + i\sin\theta) \qquad r - \text{modulus}, \ \theta - \text{amplitude}$$

Multiplication and division in polar form

$$\left[r_1(\cos\theta_1 + i\sin\theta_1)\right] \cdot \left[r_2(\cos\theta_2 + i\sin\theta_2)\right] = r_1 r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right]$$

$$\frac{r_1(\cos\theta_1 + \sin\theta_1)}{r_2(\cos\theta_2 + \sin\theta_2)} \cdot = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + \sin(\theta_1-\theta_2)\right]$$

De Moivre's theorem

$$\left[r(\cos\theta + \sin\theta)\right]^n = r^n(\cos n\theta + \sin n\theta)$$

Roots of complex numbers

$$\left[r(\cos\theta + \sin\theta)\right]^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\frac{\theta+2k\pi}{n} + \sin\frac{\theta+2k\pi}{n}\right)$$

From this the n nth roots can be obtained by putting k = 0, 1, 2, . . ., n - 1

4. Factoring and product

Factoring Formulas

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

$$a^3 + b^3 = (a+b)(a^2-ab+b^2)$$

$$a^4 - b^4 = (a-b)(a+b)(a^2+b^2)$$

$$a^5 - b^5 = (a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)$$

Product Formulas

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

$$(a+b+c+\ldots)^2 = a^2 + b^2 + c^2 + \ldots 2(ab+ac+bc+\ldots)$$

5. Algebric equations

Quadric Eqation: $ax^2 + bx + c = 0$

Solutions (roots):

$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

if $D=b^2-4ac$ is the discriminant, then the roots are

(i) real and unique if D > 0

(ii) real and equal if D = 0

(iii) complex conjugate if D < 0

Cubic Eqation: $x^3+a_1x^2+a_2x+a_3=0$

Let

$$Q=\frac{3a_2-a_1^2}{9}, \qquad R=\frac{9a_1a_2-27a_3-2a_1^3}{54}$$

$$S=\sqrt[3]{R+\sqrt{Q^3+R^2}}, \qquad T=\sqrt[3]{R-\sqrt{Q^3+R^2}}$$

then solutions are:

$$x_1=S+T-\frac{1}{3}a_1$$

$$x_2=-\frac{1}{2}(S+T)-\frac{1}{3}a_1+\frac{1}{2}i\sqrt{3}(S-T)$$

$$x_3=-\frac{1}{2}(S+T)-\frac{1}{3}a_1-\frac{1}{2}i\sqrt{3}(S-T)$$

if $D = Q^3 + R^3$ is the discriminant, then:
(i) one root is real and two complex conjugate if $D > 0$
(ii) all roots are real and at last two are equal if $D = 0$
(iii) all roots are real and unequal if $D < 0$

Cuadric Eqation: $x^4+a_1x^3+a_2x^2+a_3x+a_4=0$

Let y1 be a real root of the cubic equation

$$y^3-a_2y^2+(a_1a_3-4a_4)y+\left(4a_2a_4-a_3^2-a_1^2a_4\right)=0$$

Solution are the 4 roots of

$$z^2+\frac{1}{2}\left(a_1\pm\sqrt{a_1^2-4a_2+4y_1}\right)z+\frac{1}{2}\left(y_1\pm\sqrt{y_1^2-4a_4}\right)=0$$