Math Formulas: Triangles in two dimensions

Area of the triangle

The area of the triangle formed by the three lines:

1.
$$\begin{aligned} A_1x + B_1y + C_1 &= 0 \\ A_2x + B_2y + C_2 &= 0 \\ A_3x + B_3y + C_3 &= 0 \end{aligned}$$

is given by

2.
$$A = \frac{\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}^2}{2 \cdot \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix} \cdot \begin{vmatrix} A_2 & B_2 \\ A_3 & B_3 \end{vmatrix} \cdot \begin{vmatrix} A_3 & B_3 \\ A_1 & B_1 \end{vmatrix}}$$

The area of a triangle whose vertices are $P_1(x_1, y_1), P_2(x_2, y_2)$and $P_3(x_3, y_3)$ is given by :

3.
$$A = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

and by:

4.
$$A = \frac{1}{2} \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix}$$

Centroid

The **centroid** of a triangle whose vertices are $P_1(x_1, y_1), P_2(x_2, y_2)$ and $P_3(x_3, y_3)$ is given by:

5.
$$(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Incenter

The **incenter** of a triangle whose vertices are $P_1(x_1, y_1), P_2(x_2, y_2)$ and $P_3(x_3, y_3)$ is given by:

6.
$$(x, y) = \left(\frac{a\,x_1 + b\,x_2 + c\,x_3}{3}, \frac{a\,y_1 + b\,y_2 + c\,y_3}{3} \right)$$

where a is the length of P_2P_3, b is the length of P_3P_1, and c is the length of P_1P_2.

Circumcenter

The **circumcenter** of a triangle whose vertices are $P_1(x_1, y_1), P_2(x_2, y_2)$ and $P_3(x_3, y_3)$ is given by:

7.
$$(x, y) = \left(\frac{\begin{vmatrix} x_1^2 + y_1^2 & y_1 & 1 \\ x_2^2 + y_2^2 & y_2 & 1 \\ x_3^2 + y_3^2 & y_3 & 1 \end{vmatrix}}{2 \cdot \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}, \frac{\begin{vmatrix} x_1 & x_1^2 + y_1^2 & 1 \\ x_2 & x_2^2 + y_2^2 & 1 \\ x_3 & x_3^2 + y_3^2 & 1 \end{vmatrix}}{2 \cdot \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}} \right)$$

Orthocenter

The **orthocenter** of a triangle whose vertices are $P_1(x_1, y_1), P_2(x_2, y_2)$ and $P_3(x_3, y_3)$ is given by:

8. $$(x, y) = \left(\frac{\begin{vmatrix} y_1 & x_2x_3 + y_1^2 & 1 \\ y_2 & x_3x_1 + y_2^2 & 1 \\ y_3 & x_1x_2 + y_3^2 & 1 \end{vmatrix}}{2 \cdot \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}, \frac{\begin{vmatrix} x_1^2 + y_2y_3 & x_1 & 1 \\ x_2^2 + y_3y_1 & x_2 & 1 \\ x_3^2 + y_1y_2 & x_3 & 1 \end{vmatrix}}{2 \cdot \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}} \right)$$