Math Formulas: Planes in three dimensions

Plane forms

Point direction form:

1. $$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$$

where $P(x_1,y_1,z_1)$ lies in the plane, and the direction (a,b,c) is normal to the plane.

General form:

2. $$Ax+By+Cz+D=0$$

where direction (A,B,C) is normal to the plane.

Intercept form:

3. $$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

this plane passes through the points $(a,0,0),(0,b,0)$ and $(0,0,c)$.

Three point form:

4. $$\begin{vmatrix} x-x_3 & y-y_3 & z-z_3 \\ x_1-x_3 & y_1-y_3 & z_1-z_3 \\ x_2-x_3 & y_2-y_3 & z_2-z_3 \end{vmatrix}=0$$

Normal form:

5. $$x\cos\alpha+y\cos\beta+z\cos\gamma=p$$

Parametric form:

6. $$\begin{aligned} x&=x_1+a_1\,s+a_2\,t \\ y&=y_1+b_1\,s+b_2\,t \\ z&=z_1+c_1\,s+c_2\,t \end{aligned}$$

where the directions (a_1,b_1,c_1) and (a_2,b_2,c_2) are parallel to the plane.

Angle between two planes:

The angle between planes $A_1x+B_1y+C_1z+D_1=0$ and $A_2x+B_2y+C_2z+D_2=0$ is:

7. $$\alpha=\arccos\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\cdot\sqrt{A_2^2+B_2^2+C_2^2}}$$

The planes are parallel if and only if

8. $$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$$

Equation of a plane

The equation of a plane through $P_1(x_1,y_1,z_1)$ and parallel to directions (a_1,b_1,c_1) and (a_2,b_2,c_2) has an equation:

9. $$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

The equation of a plane through $P_1(x_1, y_1, z_1)$ and$P_1(x_2, y_2, z_2))$, and parallel to direction (a, b, c), has equation

10. $$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a & b & c \end{vmatrix} = 0$$

The equation of a plane through $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$ and $P_3(x_3, y_3, z_3)$, has equation

11. $$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Distance from point to plane

The distance of $P_1(x_1, y_1, z_1)$ from the plane $Ax + By + Cz + D = 0$ is

12. $$d = \frac{Ax_1 + By_1 + Cz_1}{\sqrt{A^2 + B^2 + C^2}}$$

Intersection of two planes

The intersection of planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is the line:

13. $$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

where

14. $$a = \begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix} \quad b = \begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix} \quad c = \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}$$
$$x_1 = \frac{b\begin{vmatrix} D_1 & C_1 \\ D_2 & C_2 \end{vmatrix} - c\begin{vmatrix} D_1 & B_1 \\ D_2 & B_2 \end{vmatrix}}{a^2 + b^2 + c^2}$$
$$y_1 = \frac{c\begin{vmatrix} D_1 & A_1 \\ D_2 & A_2 \end{vmatrix} - a\begin{vmatrix} D_1 & C_1 \\ D_2 & C_2 \end{vmatrix}}{a^2 + b^2 + c^2}$$
$$z_1 = \frac{a\begin{vmatrix} D_1 & B_1 \\ D_2 & B_2 \end{vmatrix} - b\begin{vmatrix} D_1 & A_1 \\ D_2 & A_2 \end{vmatrix}}{a^2 + b^2 + c^2}$$

If $a = b = c = 0$, then the planes are parallel.