Math Formulas: Lines in two dimensions

Line forms

Slope y-intercept form:

1. $$y = mx + b$$

Two point form:

2. $$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Point slope form:

3. $$y - y_1 = m(x - x_1)$$

Intercept form

4. $$\frac{x}{a} + \frac{y}{b} = 1 \ , \ (a, b \neq 0)$$

Normal form:

5. $$x \cdot \cos\Theta + y \cdot \sin\Theta = p$$

Parametric form:

6. $$\begin{aligned} x &= x_1 + t \cdot \cos\alpha \\ y &= y_1 + t \cdot \sin\alpha \end{aligned}$$

Point direction form:

7. $$\frac{x - x_1}{A} = \frac{y - y_1}{B}$$

where (A, B) is the direction of the line and $P_1(x_1, y_1)$ lies on the line.

General form:

8. $$Ax + By + C = 0 \ , \ (A \neq 0 \text{ or } B \neq 0)$$

Distance

The distance from $A\,x + B\,y + C = 0$ to $P_1(x_1, y_1)$ is

9. $$d = \frac{|A\,x_1 + B\,y_1 + C|}{\sqrt{A^2 + B^2}}$$

Concurrent lines

Three lines

10. $$\begin{aligned} A_1x + B_1y + C_1 &= 0 \\ A_2x + B_2y + C_2 &= 0 \\ A_3x + B_3y + C_3 &= 0 \end{aligned}$$

are concurrent if and only if:

11. $$\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = 0$$

Line segment

A line segment P_1P_2 can be represented in parametric form by

12. $$\begin{aligned} x &= x_1 + (x_2 - x_1)t \\ y &= y_1 + (y_2 - y_1)t \\ 0 &\leq t \leq 1 \end{aligned}$$

Two line segments P_1P_2 and P_3P_4 intersect if any only if the numbers

13. $$s = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix}}{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_4 & y_3 - y_4 \end{vmatrix}} \quad \text{and} \quad t = \frac{\begin{vmatrix} x_3 - x_1 & y_3 - y_1 \\ x_3 - x_4 & y_3 - y_4 \end{vmatrix}}{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_4 & y_3 - y_4 \end{vmatrix}}$$

satisfy $0 \leq s \leq 1$ and $0 \leq t \leq 1$.