Math Formulas: Definite integrals of trig functions

Note: In the following formulas all letters are positive.

Basic formulas

1. $$\int_0^{\pi/2} \sin^2 x\,dx = \int_0^{\pi/2} \cos^2 x\,dx = \frac{\pi}{4}$$

2. $$\int_0^{\infty} \frac{\sin(px)}{x}\,dx = \begin{cases} \pi/2 & p > 0 \\ 0 & p = 0 \\ -\pi/2 & p < 0 \end{cases}$$

3. $$\int_0^{\infty} \frac{\sin^2 px}{x^2} = \frac{\pi\, p}{2}$$

4. $$\int_0^{\infty} \frac{1-\cos(px)}{x^2}dx = \frac{\pi\, p}{2}$$

5. $$\int_0^{\infty} \frac{\cos(px)-\cos(qx)}{x}dx = ln\frac{q}{p}$$

6. $$\int_0^{\infty} \frac{\cos(px)-\cos(qx)}{x^2}dx = \frac{\pi(q-p)}{2}$$

7. $$\int_0^{2\pi} \frac{dx}{a+b\,\sin x} = \frac{2\pi}{\sqrt{a^2-b^2}}$$

8. $$\int_0^{2\pi} \frac{dx}{a+b\,\cos(x)} = \frac{2\pi}{\sqrt{a^2-b^2}}$$

9. $$\int_0^{\infty} \sin ax^2\,dx = \int_0^{\infty} \cos(ax^2)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{2a}}$$

10. $$\int_0^{\infty} \frac{\sin x}{\sqrt{x}}dx = \int_0^{\infty} \frac{\cos x}{\sqrt{x}}dx = \sqrt{\frac{\pi}{2}}$$

11. $$\int_0^{\infty} \frac{\sin^3 x}{x^3}dx = \frac{3\pi}{8}$$

12. $$\int_0^{\infty} \frac{\sin^4 x}{x^4}dx = \frac{\pi}{3}$$

13. $$\int_0^{\infty} \frac{\tan x}{x}dx = \frac{\pi}{2}$$

14. $$\int_0^{\pi/2} \frac{dx}{a+b\,\cos x} = \frac{\arccos(b/a)}{\sqrt{a^2-b^2}}$$

Advanced formulas

15. $$\int_0^{\pi} \sin(mx)\cdot\sin(nx)\,dx = \begin{cases} 0 & m,n \text{ integers and } m \neq n \\ \pi/2 & m,n \text{ integers and } m = n \end{cases}$$

16. $$\int_0^{\pi} \cos(mx)\cdot\cos(nx)\,dx = \begin{cases} 0 & m,n \text{ integers and } m \neq n \\ \pi/2 & m,n \text{ integers and } m = n \end{cases}$$

17. $$\int_0^{\pi} \sin(mx)\cdot\cos(nx)\,dx = \begin{cases} 0 & m,n \text{ integers and } m+n \text{ odd} \\ 2m/(m^2-n^2) & m,n \text{ integers and } m+n \text{ even} \end{cases}$$

18. $$\int_0^{\pi/2} \sin^{2m} x\,dx = \int_0^{\pi/2} \cos^{2m} x\,dx = \frac{1\cdot 3\cdot 5\ldots 2m-1}{2\cdot 4\cdot 6\ldots 2m}\frac{\pi}{2}$$

19. $$\int_0^{\pi/2} \sin^{2m+1} x\, dx = \int_0^{\pi/2} \cos^{2m+1} x\, dx = \frac{2 \cdot 4 \cdot 6 \ldots 2m}{1 \cdot 3 \cdot 5 \ldots 2m+1}$$

20. $$\int_0^{\pi} \sin^{2p-1} x\, \cos^{2q-1} x\, dx = \frac{\Gamma(p)\, \Gamma q}{2\, \Gamma(p+q)}$$

21. $$\int_0^{\infty} \frac{\sin(px) \cdot \cos(qx)}{x}\, dx = \begin{cases} 0 & p > q > 0 \\ \pi/2 & 0 < p < q \\ \pi/4 & p = q > 0 \end{cases}$$

22. $$\int_0^{\infty} \frac{\sin(px) \cdot \sin(qx)}{x^2}\, dx = \begin{cases} \pi\, p/2 & 0 < p \le q \\ \pi\, q/2 & p \ge q > 0 \end{cases}$$

23. $$\int_0^{\infty} \frac{\cos(mx)}{x^2 + a^2} dx = \frac{\pi}{2a} e^{-ma}$$

24. $$\int_0^{\infty} \frac{x\, \sin(mx)}{x^2 + a^2} dx = \frac{\pi}{2} e^{-ma}$$

25. $$\int_0^{\infty} \frac{\sin(mx)}{x\,(x^2 + a^2)} dx = \frac{\pi}{2a^2} \left(1 - e^{-ma}\right)$$

26. $$\int_0^{2\pi} \frac{dx}{(a + b\, \sin x)^2} = \int_0^{2\pi} \frac{dx}{(a + b\, \cos x)^2} = \frac{2\pi\, a}{(a^2 - b^2)^{3/2}}$$

27. $$\int_0^{2\pi} \frac{dx}{1 - 2a\, \cos x + a^2} = \frac{2\pi}{1 - a^2}, \quad 0 < a < 1$$

28. $$\int_0^{\pi} \frac{x\, \sin x\, dx}{1 - 2a\, \cos x + a^2} = \begin{cases} \frac{\pi}{a} ln(1 + a) & |a| < 1 \\ \pi\, ln(1 + \frac{1}{a}) & |a| > 1 \end{cases}$$

29. $$\int_0^{\pi} \frac{\cos(mx)\, dx}{1 - 2a\, \cos x + a^2} = \frac{\pi a^m}{1 - a^2}, \quad a^2 < 1$$

30. $$\int_0^{\infty} \sin(ax^n)\, dx = \frac{1}{na^{1/n}} \Gamma(1/n)\, \sin \frac{\pi}{2n}, \quad n > 1$$

31. $$\int_0^{\infty} \cos(ax^n)\, dx = \frac{1}{na^{1/n}} \Gamma(1/n)\, \cos \frac{\pi}{2n}, \quad n > 1$$

32. $$\int_0^{\infty} \frac{\sin x}{x^p} dx = \frac{\pi}{2\, \Gamma(p)\, \sin(p\pi/2)}, \quad 0 < p < 1$$

33. $$\int_0^{\infty} \frac{\cos x}{x^p} dx = \frac{\pi}{2\, \Gamma(p)\, \cos(p\pi/2)}, \quad 0 < p < 1$$

34. $$\int_0^{\infty} \sin(ax^2)\, \cos(2bx)\, dx = \frac{1}{2} \sqrt{\frac{\pi}{2a}} \left(\cos \frac{b^2}{a} - \sin \frac{b^2}{a}\right)$$

35. $$\int_0^{\infty} \cos(ax^2)\, \cos(2bx)\, dx = \frac{1}{2} \sqrt{\frac{\pi}{2a}} \left(\cos \frac{b^2}{a} + \sin \frac{b^2}{a}\right)$$

36. $$\int_0^{\infty} \frac{dx}{1 + \tan^m x} dx = \frac{\pi}{4}$$