Math Formulas: Definite integrals of logarithmic functions

1. $$\int_0^1 x^m (\ln x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}, \quad m > -1,\ n = 0, 1, 2, \ldots$$

2. $$\int_0^1 \frac{\ln x}{1+x} dx = -\frac{\pi^2}{12}$$

3. $$\int_0^1 \frac{\ln x}{1-x} dx = -\frac{\pi^2}{6}$$

4. $$\int_0^1 \frac{\ln(1+x)}{x} dx = \frac{\pi^2}{12}$$

5. $$\int_0^1 \frac{\ln(1-x)}{x} dx = -\frac{\pi^2}{6}$$

6. $$\int_0^1 \ln x \ln(1+x)\, dx = 2 - 2\ln 2 - \frac{\pi^2}{12}$$

7. $$\int_0^1 \ln x \ln(1-x)\, dx = 2 - \frac{\pi^2}{6}$$

8. $$\int_0^\infty \frac{x^{p-1}\ \ln x}{1+x} dx = -\pi^2\ \csc(p\pi)\ \cot(p\pi), 0 < p < 1$$

9. $$\int_0^1 \frac{x^m - x^n}{\ln x} dx = \ln \frac{m+1}{n+1}$$

10. $$\int_0^\infty e^{-x}\ \ln x\, dx = -\gamma$$

11. $$\int_0^\infty e^{-x^2} \ln x\, dx = -\frac{\sqrt{\pi}}{4}(\gamma + 2\ \ln 2)$$

12. $$\int_0^\infty \ln\left(\frac{e^x+1}{e^x-1}\right) dx = \frac{\pi^2}{4}$$

13. $$\int_0^{\pi/2} \ln(\sin x) dx = \int_0^{\pi/2} \ln(\cos x) dx = -\frac{\pi}{2}\ln 2$$

14. $$\int_0^{\pi/2} (\ln(\sin x))^2 dx = \int_0^{\pi/2} (\ln(\cos x))^2 dx = \frac{\pi}{2}(\ln 2)^2 + \frac{\pi^3}{24}$$

15. $$\int_0^\pi x\ \ln(\sin x) dx = -\frac{\pi^2}{2}\ \ln 2$$

16. $$\int_0^{\pi/2} \sin x \ln(\sin x) dx = \ln 2 - 1$$

17. $$\int_0^{2\pi} \ln(a + b\sin x) dx = \int_0^{2\pi} \ln(a + b\cos x) dx = 2\pi \ln\left(a + \sqrt{a^2 - b^2}\right)$$

18. $$\int_0^\pi \ln(a + b\cos x) dx = \pi \ln\left(\frac{a + \sqrt{a^2 - b^2}}{2}\right)$$

19. $$\int_0^\pi \ln\left(a^2 - 2ab\cos x + b^2\right) dx = \begin{cases} 2\pi \ln a & a \geq b > 0 \\ 2\pi \ln b & b \geq a > 0 \end{cases}$$

20. $$\int_0^{\pi/4} \ln(1 + \tan x) dx = \frac{\pi}{8}\ln 2$$

21. $$\int_0^{\frac{\pi}{2}} \sec x\ \ln\left(\frac{1 + b\cos x}{1 + a\cos x}\right) dx = \frac{1}{2}\left(\arccos^2 a - \arccos^2 b\right)$$