Math Formulas: Lines in three dimensions

Line forms

Point direction form:

1. $$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

Two point form:

2. $$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

Parametric form:

3. $$\begin{aligned} x &= x_1 + t\,\cos\alpha \\ y &= y_1 + t\,\cos\beta \\ z &= z_1 + t\,\cos\gamma \end{aligned}$$

Distance between two lines in 3 dimensions

The **distance** from $P_2(x_2, y_2, z_2)$ to the line through $P_1(x_1, y_1, z_1)$ in the direction (a, b, c) is

4. $$d=\sqrt{\frac{[c(y_2-y_1)-b(z_2-z_1)]^2+[a(z_2-z_1)-c(x_2-x_1)]^2+[b(x_2-x_1)-a(y_2-y_1)]^2}{a^2+b^2+c^2}}$$

The **distance** between two lines. First one through $P_1(x_1, y_1, z_1)$ in direction (a_1, b_1, c_1), Second one: through $P_2(x_2, y_2, z_2)$ in direction (a_2, b_2, c_2) is:

5. $$d=\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}^2+\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}^2+\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}^2}}$$

The two lines intersect if:

6. $$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$